

2004 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 1-2

Name \_\_\_\_\_

Time Limit 9 Minutes

School \_\_\_\_\_

1. Find the maximum area of a triangle in which the length of one side is 6 and the sum of the lengths of the other sides is 10.

2. Find all ordered triples of positive integers  $(x,y,z)$  such that  $\frac{1}{x + \frac{1}{y + \frac{1}{z}}} = \frac{7}{10}$

ANSWER TO PROBLEM 1

ANSWER TO PROBLEM 2

2004 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 3-4

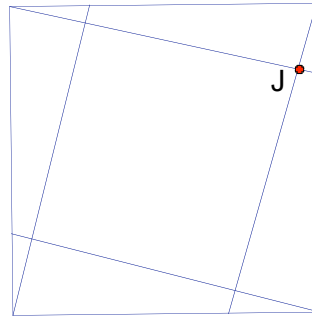
Name \_\_\_\_\_

Time Limit 10 Minutes

School \_\_\_\_\_

3. ABCD is an isosceles trapezoid with  $AB = 10$  and  $CD = 8$ . E is the midpoint of AB and F is the midpoint of CD, with  $EF = 12$ . Point R is on segment EF such that  $\angle CRB$  is a right angle. Find all possible lengths for RE.

4. In the figure, square ABCD is a square of side 12. Points E, F, G, and H are trisection points of the sides AB, BC, CD, and DA nearer A, B, C, and D, respectively. Find the area of quadrilateral IJKL.



ANSWER TO PROBLEM 3

ANSWER TO PROBLEM 4

2004 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 5-6

Name \_\_\_\_\_

Time Limit 10 Minutes

School \_\_\_\_\_

5. Find all positive real numbers  $x$  such that  $\frac{1}{x + \sqrt{x}} + \frac{1}{x - \sqrt{x}} \leq 1$

6. A, B, and C are points on a circle of radius 3 with  $\angle ACB = 30^\circ$ . If  $AC = 2$ , find length BC.

ANSWER TO PROBLEM 5

ANSWER TO PROBLEM 6

**2004 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS**

**Problems 7-8**

**Name** \_\_\_\_\_

**Time Limit 12 Minutes**

**School** \_\_\_\_\_

7. Given that  $x^3 + \frac{1}{x^3} = 110$ , find all possible non-real values for  $x + \frac{1}{x}$

8. Find all ordered pairs of integers  $(x,y)$  such that  $1 + 2x + 3y = xy$

**ANSWER TO PROBLEM 7**

**ANSWER TO PROBLEM 8**

2004 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 9-10

Name \_\_\_\_\_

Time Limit 11 Minutes

School \_\_\_\_\_

9. Stan is standing still. At a given signal, Stan either takes a step left (with probability  $\frac{1}{3}$ ), a step right (with probability  $\frac{1}{6}$ ), or stands still (with probability  $\frac{1}{2}$ ). The signal is given four times. Find the probability that Stan ends up standing in the same place where he was standing still.

10. Evaluate  $(\sin 50^\circ)(\sin 170^\circ)(\sin 250^\circ)$

ANSWER TO PROBLEM 9

ANSWER TO PROBLEM 10

2004 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 11-12

Name \_\_\_\_\_

Time Limit 10 Minutes

School \_\_\_\_\_

11. At the law firm of Dewey, Cheatem, and Howe, one of the three partners has committed a crime. When questioned by the police, each gave two statements. The two innocent men each gave one true statement and one false statement. The guilty person gave two false statements. Their statements were: Dewey: (1) Cheatem and Howe are both guilty; (2) If Cheatem is guilty, then Howe is innocent. Cheatem: (1) Howe is innocent and I'm innocent. (2) If Dewey is guilty, then Howe is guilty. Howe: (1) Dewey or Cheatem did it. (2) Dewey's second statement is false. Who is the guilty party?
12. Suppose that, in any one game of a fantasy baseball league, the probability that the Cubs beat the Sox is  $\frac{2}{5}$ . Find the probability that the Cubs win the World Series 4 games to 2. [For those unfamiliar with baseball, no games end in ties, and the World Series ends when one team has won four games]

ANSWER TO PROBLEM 11

ANSWER TO PROBLEM 12

2004 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 13-14

Name \_\_\_\_\_

Time Limit 10 Minutes

School \_\_\_\_\_

13. The perimeter of a rhombus is 24 and the area of the rhombus is 13, find the sum of the diagonals of the rhombus

14. In the alphametic multiplication problem shown at right, each letter stands for a unique digit. What digit does the letter C stand for?

$$\begin{array}{r} ABCDE \\ \times \quad E \\ \hline EDADE \end{array}$$

ANSWER TO PROBLEM 13

ANSWER TO PROBLEM 14

2004 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 15-16

Name \_\_\_\_\_

Time Limit 10 Minutes

School \_\_\_\_\_

15. If  $24^x \cdot 18^y = 6$ , find the ordered pair of rational numbers (x,y)

16. Angle A is acute with  $\sin A = \frac{24}{25}$ . Find the value of  $\sin\left(\frac{A}{2}\right)$

ANSWER TO PROBLEM 15

ANSWER TO PROBLEM 16

2004 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 17-18

Name \_\_\_\_\_

Time Limit 11 Minutes

School \_\_\_\_\_

17. Find the number base in which the numbers 4, 11, 27 form a geometric sequence.

18. Complete the cross-number puzzle at right in which each across answer is a four-digit positive integer (i.e. an integer from 1000 to 9999) and each down answer is a three-digit positive integer (i.e. an integer from 100 to 999).

1	2	3	4
5			
6			

[Note: the grid at right is for scratch work only. The answer must appear in the answer space at the bottom of the page.]

Across

1. A perfect Square
5. A Fibonacci Number
6. The average of all 4-digit even palindromes which are multiples of 9

Down

1. A perfect square
2. Two less than a perfect square
3. A perfect square
4. Digits in strictly increasing order

ANSWER TO PROBLEM 17

ANSWER TO PROBLEM 18

1	2	3	4
5			
6			

2004 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 19-20

Name \_\_\_\_\_

Time Limit 13 Minutes

School \_\_\_\_\_

19. In an ellipse,  $M$  and  $N$  are the foci with length  $MN = 4$ . Points A and B are on the ellipse, with  $AB = 2\sqrt{13}$ , such that  $AB$  is not parallel to  $MN$ ,  $AM \perp MN$ , and  $BN \perp MN$ , Find the eccentricity of the ellipse.

20. If  $r$  and  $s$  are the solutions to the equation  $\tan^2 x - 5 \tan x + 2 = 0$ , with  $r$  and  $s$  both between  $0$  and  $\pi$ , find the value of  $\tan(r + s)$ .

ANSWER TO PROBLEM 19

ANSWER TO PROBLEM 20

2004 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 21-22

Name \_\_\_\_\_

Time Limit 11 Minutes

School \_\_\_\_\_

21. In a circle, chord  $AB$  of length 2 and chord  $AC$  of length  $5\sqrt{3}$  meet at a  $30^\circ$  angle. Find the area of the circle.
22. In parallelogram  $ABCD$ ,  $\angle A$  is acute and  $AB = 5$ . Point  $E$  is on  $\overline{AD}$  with  $AE = 4$ ,  $BE = 3$ . A line through  $B$ , perpendicular to  $\overline{CD}$ , intersects line  $\overleftrightarrow{CD}$  at  $F$ . If  $BF = 5$ , find length  $EF$ .

ANSWER TO PROBLEM 21

ANSWER TO PROBLEM 22

2004 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

Problems 23-24

Name \_\_\_\_\_

Time Limit 10 Minutes

School \_\_\_\_\_

23. The roots of the polynomial equation  $2x^3 + 6x^2 + kx - 4 = 0$  are in arithmetic progression. Find  $k$ .
24. Angle  $ACB$  is a right angle. On ray  $CB$  are a sequence of points  $B_1, B_2, B_3, \dots$  increasingly distant from point  $C$  and so that  $CB_1 = B_1B_2 = B_2B_3 = \dots$ . Segment  $AB_3$  bisects angle  $CAB_9$ . Find the measure of angle  $AB_3B_9$ .

ANSWER TO PROBLEM 23

ANSWER TO PROBLEM 24

**2004 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS  
ANSWERS 1-14**

1. 12

2. (1,2,3)

3. 2, 10

4.  $\frac{288}{5}$

5.  $0 < x < 1$  or  $x \geq 3$

6.  $2\sqrt{2} + \sqrt{3}$

7.  $\frac{-5}{2} + \frac{3i\sqrt{7}}{2}$ ,  $\frac{-5}{2} - \frac{3i\sqrt{7}}{2}$

8. (4,9), (10,3), (2,-5), (-4,1)

9.  $\frac{107}{432}$

10.  $\frac{-1}{8}$

11. Howe

12.  $\frac{288}{3125}$

13. 14

14. 4

2004 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS  
ANSWERS 15-24

15.  $\left(\frac{1}{5}, \frac{2}{5}\right)$

20. -5

16.  $\frac{3}{5}$

21.  $49\pi$

17. 9

22.  $\sqrt{10}$

18.  $\begin{array}{cccc} 6 & 2 & 4 & 1 \\ 2 & 5 & 8 & 4 \\ 5 & 4 & 4 & 5 \end{array}$

23. 0

19.  $\frac{1}{2}$

24. 120